Int. J. Heat Mass Transfer. Vol. 17, pp. 1129–1134. Pergamon Press 1964. Printed in Great Britain

SEMI-EMPIRICAL METHOD OF DETERMINING THE HEAT-TRANSFER COEFFICIENT FOR SUBCOOLED, SATURATED BOILING IN A CHANNEL

J. MIKIELEWICZ
Institute of Fluid-Flow Machines, Polish Academy of Sciences, Gdańsk, Poland

(*Received* 8 *February* 1973)

Abstract—On the assumption that heat transfer during flow boiling is characterized by the resultant dissipation of the energy of convective flow of a two-phase mixture and of vapour bubble generation, a relation has been derived defining the heat-transfer coefficient for a subcooled and saturated boiling in a channel. The relation obtained has been compared with the results of experiments of the present and other authors.

NOMENCLATURE

A, B, C, constants;
d, channel diameter;
E, energy;
l, length;
p, pressure;
R, friction coefficient of a two-phase mixture;
t, temperature;
v, specific volume;
w, velocity;
x, quality;
Re, Reynolds number.

Greek symbols
α, heat-transfer coefficient;
ζ, friction coefficient of single-phase flow;
μ, dynamic viscosity;
ν, kinematic viscosity;
ρ, density;
τ, shear stress at the wall;
φ, void fraction.

Subscripts
j, homogeneous mixture;
l, liquid phase;
pb, pool boiling;
TP, two-phase flow;
TPb, two-phase flow boiling;
tt, Martinelli parameter.

1. INTRODUCTION

BOILING at low flow velocities generated by natural convection, i.e. so called "pool boiling", is a process known sufficiently well nowadays. It has, however, limited practical applications. With this kind of boiling the hydrodynamics of flow does not influence markedly the heat-transfer process. The effect of the flow hydrodynamics becomes important at higher flow velocities and qualities. The formation of vapour phase during flow boiling generates various two-phase flow patterns depending on the velocity, the vapour phase content and also on the situation of the channel axis relative to the direction of gravity. Different flow patterns are connected with different heat-transfer mechanisms. The problem is too difficult to be dealt with theoretically, therefore, theoretical works deal with the heat-transfer mechanism for only some of the patterns of the two-phase flow, often only qualitatively. The existing experimental data are mainly concerned with water; very rarely are they devoted to other media. The formulas derived from experiments are not of a general character, they are valid only within the examined range of variation of the dimensionless parameters describing the process of boiling. The formulas and data for flow boiling encountered in the literature do not always coincide.

A semi-empirical method of approach to the problem of heat transfer during flow boiling is proposed in this paper. The known experimental quantities characteristic of flow boiling have been introduced into the theoretical analysis, namely: the resistance coefficient of two-phase flow (characteristic of the flow hydrodynamics) and the heat transfer coefficient for negligible flow velocity during boiling (i.e. for "pool boiling"). Based on the knowledge of those two quantities, a method is given for determining the heat-transfer coefficient for the case of complex flow boiling, both the subcooled and the saturated one.

2. GENERAL RELATION DESCRIBING FLOW BOILING IN A CHANNEL

Let us start with the simpler problem of heat transfer in a two-phase flow where there is no generation of vapour bubbles [1].

The pressure loss in the two-phase flow may be written as

$$\Delta p_{TP} = R\Delta p_0 \quad (1)$$

where Δp_0 denotes the loss of pressure in the liquid phase flowing within the total amount of two-phase flow.

The loss of pressure in two-phase flow may also be regarded as a resulting loss in a hypothetical equivalent flow of liquid phase in the same channel

$$\Delta p_{TP} = \frac{l}{d}\zeta_{TP}\rho_l\frac{w_{TP}^2}{2} \quad (2)$$

with: w_{TP}-liquid phase flow velocity.

Since there is

$$\Delta p_0 = \frac{l}{d}\zeta_0\frac{\rho_l w_0^2}{2} \quad (3)$$

and since it is known from a relation given by K_α that

$$\zeta_{TP} = CRe_{TP}^{-0.2}, \quad \zeta_0 = CRe_0^{-0.2}$$

we obtain from (1) (2) (3), that:

$$w_{TP} = R^{0.55}w_0 \approx R^{0.5}w_0. \quad (4)$$

Replacing the two-phase flow with the equivalent flow of liquid phase we may consider that the conventional heat transfer in this flow is described by relations valid for single-phase flow. Introducing (4) into the Reynolds number one obtains

$$\frac{\alpha_{TP}}{\alpha_0} = R^{0.4} \quad (5)$$

where α_0-heat-transfer coefficient for the flow of liquid with flow rate equal to that of the two-phase flow.

The above result may be employed in the description of a more complicated case of heat transfer in a two-phase flow with vapour bubble generation [2].

Let us assume that the heat transfer during flow boiling is characterized by the total dissipation of energy of the convective two-phase flow E_{TP}, and of vapour bubble generation in the flow E_{pb} [3],

$$E_{TPb} = E_{TP} + E_{pb}. \quad (6)$$

The rate of dissipation of energy in a steady flow is equal, in rough approximation, to the dissipation of energy in the boundary layer.

The dissipation of the two-phase flow energy within the volume of the boundary layer is:

$$E_{TP} = \frac{\tau_{TP}^2}{\mu_l} \quad (7)$$

but

$$\tau_{TP} = \frac{\zeta_{TP}^2}{8}\rho_l w_{TP}^2 \quad (8)$$

where

$$\zeta_{TP} = R\zeta_0.$$

From (7) and (8) it follows that

$$E_{TP} = \frac{\zeta_{TP}^2\rho_l w_{TP}^4}{64\nu_l}. \quad (9)$$

It may be taken, analogously, that there is a flow resistance coefficient for the vapour bubbles generation itself, ζ_b [4].

The dissipation of energy during generation of vapour bubbles in a two-phase flow of equivalent velocity w_{TP} is represented analogously to [9], i.e.

$$E_{pb} = \frac{\zeta_b^2\rho_l w_{TP}^4}{64\nu_l}. \quad (10)$$

Writing down similarly the total energy dissipated in a two-phase flow with bubble generation

$$E_{TPb} = \frac{\zeta_{TPb}^2\rho_l w_{TP}^4}{64\nu_l} \quad (11)$$

we obtain from (6), taking account of (9)–(11):

$$\zeta_{TPb}^2 = \zeta_{TP}^2 + \zeta_{pb}^2. \quad (12)$$

By virtue of analogy to the single-phase turbulent flow, between the transport of heat and mechanical energy, relation (12) yields:

$$\alpha_{TPb}^2 = \alpha_{TP}^2 + \alpha_{pb}^2. \quad (13)$$

Whereas, taking into account (5) there is

$$\frac{\alpha_{TPb}}{\alpha_0} = \sqrt{\left[R^{0.8} + \left(\frac{\alpha_{pb}}{\alpha_0}\right)^2\right]} \quad (14)$$

The heat-transfer coefficient for pool boiling in the above relations (use being made of the above mentioned analogy) was referred to the total temperature difference between the channel wall and the flowing medium. In case of subcooled boiling the respective coefficient of "pool" heat transfer $\alpha_{pb}(\Delta t_s)$ should, then, be referred also to the total temperature difference $\Delta t_s + \Delta t_p$.

The general relation concerning both the saturated and subcooled boiling will be as follows:

$$\frac{\alpha_{TPb}}{\alpha_0} = \sqrt{\left\{R^{0.8} + \left[\frac{\alpha_{pb}(\Delta t_s)}{\alpha_0}\cdot\frac{\Delta t_s}{\Delta t_s + \Delta t_p}\right]^2\right\}}. \quad (15)$$

Table 1. Experimental values of A and n appearing in (17), after different author

Item	Author	Fluid	A	n
1	Wright [5]	water	2·72	0·58
2	Dengler and Addoms [5]	water	3·5	0·5
3	Schrock and Grossman [5]	water	2·5	0·7
4	Collier and Bulling [5]	water	2·17	0·7
5	Guerrieri and Talty [8]	Pentone and Heptone	3·4	0·45
6	Pujol and Stennig [8]	freon	4	0·37
7	Bennett [8]	water	2·9	0·66

Some additional comments should be made on the hydraulic resistance of a two-phase mixture. There are a number of methods for calculating R [5]:

(a) analytically for "uniform" flow model, e.g.

$$R_j = 1 + \frac{v_v - v_l}{v_l} \tag{16}$$

(b) for the "slip" model the most commonly known are: generalized Lockhart–Martinelli method, Martinelli–Nelson method for water, Lottes and Flinn method with $R = (1-x/1-\varphi)^2$, Levi, Chisholm and other methods.

3. COMPARISON WITH EXPERIMENTAL DATA

In order to check the validity of equations (14) or (15), an analysis of available published experimental data on the flow boiling in a channel has been carried out.

The convective heat transfer in a two-phase flow with negligible generation of vapour bubbles is, in most cases, described by a relation of the form:

$$\frac{\alpha_{TP}}{\alpha_l} = A\left(\frac{1}{X_{tt}}\right)^n \tag{17}$$

where the values of A and n obtained from the experiments by different authors have been listed in Table 1.

Comparative calculations have been performed for the mixture of steam and water at $p = 70{\cdot}3$ ata, using these experimental relations and the relation (5): the latter having been transformed by introducing the coefficient α_l—of heat transfer of the liquid phase taken as if the liquid were flowing independently in the channel at the rate occuring in the two-phase flow. Also the function describing the convective heat transfer of the two-phase flow given graphically by Chen [5] has been used in the comparison. The results of the comparison have been presented in Fig. 1.

For low flow qualities, i.e. for $x \to 0$ and for $R \to 1$ relation (14) transforms itself into the known, experimentally established relation of Kutateladze [6]:

$$\frac{\alpha_{TP}}{\alpha_0} = \sqrt{\left[1+\left(\frac{\alpha_{pb}}{\alpha_0}\right)^2\right]}. \tag{18}$$

It must be noted that the conditions for vapour bubble generation in a two-phase mixture of a boiling medium do not always exist. It is, therefore, not always logical to include the term α_{pb}/α_0 in the above given relations.

For large void fractions, when vapour forms the flow core, the velocity at the interface between the liquid and vapour phases is very high, with resulting high flow turbulence. The mechanism of evaporation is then altered. The evaporation is effected at the interface of phases, and not at the channel wall [5].

In this situation bubble boiling is suppressed, and it is unnecessary to consider the term α_{pb}/α_0 in the above relations for calculating the heat-transfer coefficient. This problem has been discussed earlier [7]. In order to prove a generaity of the relation (15), the results calculated using this relation have been compared with the experimental results for supercooled and saturated boiling. In the calculation it was assumed that the friction losses were equal to those for homogeneous isothermal flow. The experimental results considered have been listed in Table 2; for the results of the comparison, see Fig. 2.

Experimental results under non-isothermal conditions [17–19] show that the two-phase friction factor reaches at certain qualities a "crisis", i.e. falls off rapidly. It was noticed in reference [19] that this crisis is accompanied by a similar crisis in the heat-transfer coefficient. This result is in agreement with the general analogy between energy and heat transfer. A larger friction coefficient is always accompanied by a larger heat-transfer coefficient.

It follows from the preceding analysis that the crisis in the heat-transfer coefficient may result either from a crisis in the friction factor or from a decay of bubble generation and nucleate boiling at any given fluid super-heat. Then equation (14) becomes the relation determining the critical value of the heat-transfer coefficient. This problem was discussed by the author in [20]. There it was shown that the crisis in the two-phase friction factor occurs when the isothermal friction factor is equal to the non-isothermal one. Equation (12) shown that for this equality to occur we must have $\zeta_{TPb} = \zeta_{TP}$ which means no nucleate boiling in two-phase flow.

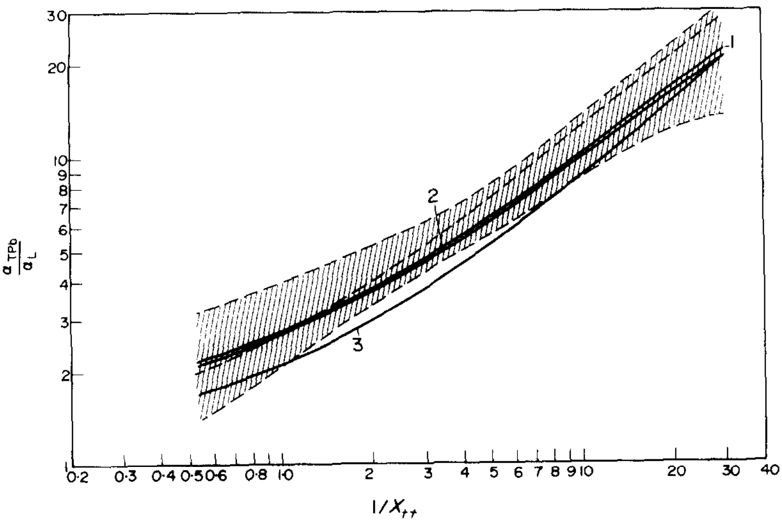

FIG. 1. Dependence of α_{TPb}/α_l on parameter $1/X_{tt}$

(1) After $\frac{\alpha_{TP}}{\alpha_l} = \left[\frac{R}{(1-X)^2}\right]^{0\cdot4}$

R after Martinelli

(2) After $\frac{\alpha_{TP}}{\alpha_l} = \left[\frac{1}{1-\varphi}\right]^{0\cdot8}$

φ after Martinelli

(3) After $\frac{\alpha_{TP}}{\alpha_l} = \left[\frac{1 + \frac{v_v - v_l}{v_l}}{(1-x)^2}\right]^{0\cdot4}$.

\\\\\\\\, Range of approximation of data correlated after Table 1

----, After Chen [5].

Table 2. Results of experiments on flow boiling used for plotting Fig. 2

Item	Author	Fluid	Measuring section geometry	Quality X	Mass velocity $w\rho$	Pressure	Heat flux	Symbol in Fig. 2
	—	—	—	—	$\left(\frac{kg}{m^2 . s}\right)$	bar	$\left(\frac{W}{m^2} \times 10^{-3}\right)$	—
1	Kowalczewski [9]	R-12	vertical pipe i.d. = 10·87 mm	0·2–0·6	360	7·45	19·2–48·1	○
2	Uchidai Yamaguchi [10]	R-12	horizontal pipe i.d. = 6·4 mm	0·2–0·9	340–510	3·9	2·4–28	△
3	Chawla [11]	R-11	horizontal pipe i.d. = 14 mm	0·2–0·9	470–1550	0·6	1·1–23	□
4	Curtis [12]	R-21	vertical pipe i.d. = 23 mm	0·2–0·65	—	17·8	10–14	●
5	Jezierski [13]	R-21	vertical pipe i.d. = 12 mm	0·05–0·5	1100–4200	13·5–15	17–80	+
6	Mikielewicz [14]	R-21	vertical pipe i.d. = 12 mm	<0	1280–4500	6·8–11·7	21–95	×
7	Altmann Norris [15] Staub	R-22	two horizontal pipes i.d. = 8·70 mm	0·3–0·9	80–560	5·5–11	5·4–36	▲
8	Murphy Bergles [16]	R-113	horizontal pipe i.d. = 11 mm	<0	500	1·54	19–57	■

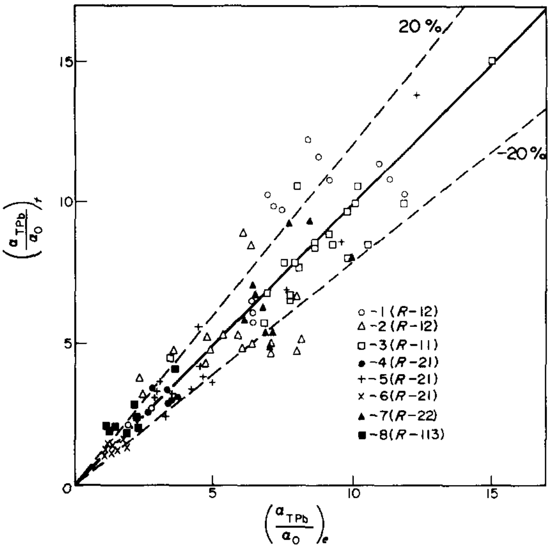

FIG. 2. Comparison between the values of heat-transfer coefficient for flow boiling calculated after relation (15) and the results of experiments listed in Table 2.

4. CONCLUSIONS

The agreement of the values calculated using the derived relations with experimental values obtained by different authors was satisfactory.

A knowledge of precise values of the heat-transfer coefficient for "pool boiling" is necessary for this method, as well as of the hydraulic resistance coefficient of the two-phase flow. Consequently, there is a need of further investigations of both quantities regarded as the basic ones characteristic of the complex process of flow boiling.

If, lacking other data, the simplest (homogeneous) approximation for the hydraulic resistance coefficient is used, this may in some cases lead to serious errors.

Relation (14) may be regarded as a method of indication for correlating the experimental results on saturated flow boiling.

For processing the experimental data on saturated boiling in the flow, the following interpolation formula suggested

$$\frac{\alpha_{TPb}}{\alpha_0} = \left[B\left(\frac{1}{X_{tt}}\right)^m + \left(\frac{\alpha_{pb}}{\alpha_0}\right)^2\right]^{0.5} \tag{19}$$

which was employed heretofore in contracted form (17), for the case when $\alpha_{pb}/\alpha_0 = 0$.

The term $B(1/X_{tt})^m$ replaces the function R, in the above relation, the function itself being dependent on X_{tt} parameter according to the Lockhart–Martinelli method.

Equation (14) allows a qualitative explanation of the boiling crisis in forced convective boiling. A quantitative formulation of the crisis requires further theoretical as well as experimental work. In particular the friction factor corresponding to film boiling and to the limit of nucleate boiling needs further experimental study.

REFERENCES

1. J. Mikielewicz, Analogy of heat and mechanical energy transport in two-phase flow in channels, *Prace IMP* No. 52, 81–89 (1971).
2. J. Mikielewicz, Selected problems of low-boiling fluid vapour generation of a binary cycle, *Prace IMP* No. 34, 720 (1972).
3. W. Damanski, Semi-theoretical solution of nucleate boiling heat transfer, *Inzh.-Fiz. Zh.* **19**(4), 629–637 (1970).
4. E. I. Ivanov, On heat-transfer mechanism with nucleate water boiling in tube and Reynolds analogy, in *Convective Heat Transfer in Two-Phase and Single-Phase Flows*. Inzd. "Energiya", Leningrad (1964).
5. L. S. Tong, *Boiling Heat Transfer and Two-Phase Flow*. John Wiley, New York (1965).
6. S. S. Kutateladze, Boiling heat and mass transfer, *Int. J. Heat Mass Transfer* **4** (1961).
7. J. Mikielewicz. Vapour phase formation during flow boiling in a vapour generator of a binary power plant, *Prace IMP* No. 56, 63–72 (1971).
8. J. M. Chawla, Correlation of convective heat transfer coefficient for two-phase liquid–vapour flow, *Proceedings 4th Int. Heat Transfer Conference, Paris*, 1970, Vol. 5.
9. H. Schicht, Experimentelle Untersuchung der Strömmung eines Flüsigkeitsdamfgemisches von R-12 einer senkrechten Rohrleitung (nach Versuchen von Dr I. I. Kowalczewski), *Kältetechnik*. **17**(2), 47–51 (1965).
10. H. Uchida and S. Yamaguchi, Heat transfer in two-phase boiling of Freon-12 in a horizontal tube, in *Advances in Heat Transfer*. Izd. "Mir", Moscow (1970).
11. I. M. Chawla, Wärmeübergang und Druckabfall in waggerechten Rohren bei der Strömmung von verdampfenden Kältemitteln, *ForschHft. Ver. Dt. Ing.* 523 (1967).
12. E. M. Curtis, Boiling heat transfer of Freon-12 in a vertical electrically heated test section, CEGB Research Report RD/M/R 96 (1968).
13. A. Jezierski, Investigation of heat transfer during flow boiling of freon-21 in a vertical pipe, in conditions corresponding to these in a binary power plant, Doctoral Thesis, Shipbuilding Dept., Technical University of Gdańsk (1973).
14. J. Mikielewicz, Heat transfer and flow resistance of freon-21 during subcooled boiling in a vapour generator channel, *Prace IMP* No. 60, 109–121 (1972).
15. M. Altmann, R. H. Norris and F. W. Staub, Local and average heat transfer and pressure drop for refrigerants evaporating in horizontal tubes, *J. Heat Transfer* **82**(3), 189–199 (1960).
16. R. W. Murphy and A. E. Bergles, Subcooled flow boiling of fluorocarbons, Rept. No. DSR 71903-72, Massachusetts Institute of Technology, Cambridge (1971).
17. N. W. Tarasova and A. I. Leontiev, Hydraulic resistance in water-steam mixture flow in a heated vertical tube, *Teplofiz. Vysok. Temp.* No. 1, 115–124 (1965).

18. N. V. Tarasova, Hydraulic resistance with boiling of water and steam-water mixture in heated tubes and circular tubes, *Trudy TsKTI*, No. 57.
19. M. M. Prizakowskii and I. N. Petrova, Hydraulic tube resistance in upward flow of vapour-steam mixture at high speeds and high and superhigh pressure, *Teploenergetika*, No. 6, 25–28 (1961).
20. J. Mikielewicz, A theoretical analysis of the boiling crisis in forced convection boiling, *Trans. Inst. Fluid Flow Machines* No. 64, 31–41 (1974).

METHODE SEMI-EMPIRIQUE DE DETERMINATION DU COEFFICIENT DE TRANSFERT POUR L'EBULLITION SATUREE ET SOUS-REFROIDIE DANS UN CANAL

Résumé—En admettant que le transfert de chaleur dans un écoulement avec ébullition est caractérisé par la dissipation de l'énergie de l'écoulement du mélange biphasique et par la génération des bulles de vapeur, on a établi une relation qui définit le coefficient de transfert thermique pour une ébullition sous-refroidie et saturée dans un canal. La relation établie a été comparée aux résultats des expériences de l'auteur et d'autres chercheurs.

HALBEMPIRISCHER ANSATZ ZUR BERECHNUNG DES WÄRMEÜBERGANGSKOEFFIZIENTEN FÜR UNTERKÜHLTE UND GESÄTTIGTE VERDAMPFUNG IN EINEM KANAL

Zusammenfassung—Für den Wärmeübergangskoeffizienten bei Verdampfung in unterkühlter und gesättigter Flüssigkeit in einem Kanal wurde eine Beziehung abgeleitet, die als Voraussetzung enthält, daß der Wärmeübergang bei Verdampfung mit erzwungener Konvektion durch die resultierende Dissipationsenergie der konvektiven Zwei-Phasen-Strömung und der Dampfblasenerzeugung bestimmt wird. Die erhaltene Beziehung wurde mit eigenen Versuchsergebnisse und denen anderer Autoren verglichen.

ПОЛУЭМПИРИЧЕСКИЙ МЕТОД ОПРЕДЕЛЕНИЯ КОЭФФИЦИЕНТА ТЕПЛООБМЕНА ПРИ НАСЫЩЕННОМ КИПЕНИИ И КИПЕНИИ С НЕДОГРЕВОМ

Аннотация — В предположении, что теплообмен при кипении характеризуется диссипацией энергии конвективного потока двухфазной смеси и образованием паровых пузырей, выводится соотношение для коэффициента теплообмена при насыщенном кипении и кипении с недогревом. Полученное соотношение сравнивается с результатами экспериментов автора и других исследователей.